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LETTER TO THE EDITOR

Numerical study of transport properties in continuum percolation

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Abstract. We present numerical simulations of AC conductance for a random resistor-capacitor network. The conductance obeys a probability density function $p(g)\propto g^{-\alpha}$ $(0<\alpha<1)$. We use a highly efficient propagation algorithm to calculate the effective conductance of a long strip of a lattice. At low frequencies, we find that for the concentration p of conducting bonds less than the percolation threshold p_c, the imaginary part of conductance is proportional to frequency $\mathrm{Im}(g_{\mathrm{eff}})\simeq\omega$ and the real part of conductance shows an anomalous frequency dependence $\mathrm{Re}(g_{\mathrm{eff}})\simeq\omega^{2-\alpha}$. The results of simulations in such a continuum system are in agreement with the predictions of the effective medium and the Maxwell-Garnett approximation. We also calculate the non-universal DC conductivity exponents in continuum percolation; the results are consistent with earlier theoretical predictions and numerical calculations.

The problem of non-universal percolation in the presence of a broad distribution of bond strength was first studied by Kogut and Straley [1] and later by Ben-Mizrahi and Bergman [2] and Straley [3]. The model treated is that of conduction in a percolating lattice where the conductance of the bonds has the probability distribution

$$p(g)=(1-p)\delta(g-g_2)+ph(g) \tag{1}$$

with $h(g)=(1-\alpha)g^{-\alpha}$ $(0<g<1)$ where $0<\alpha<1$.

Recently Halperin *et al* [4, 5] have shown that in the so-called 'swiss cheese model' of a percolating system, the transport exponents may not be universal. When this model is mapped onto a discrete network [6], it leads to a probability distribution $p(g)$ in the form of (1). It turns out [5, 7] that in such a model the conductivity exponent $\bar{t}$ is bounded by

$$\max\left(t_1+\frac{\alpha}{1-\alpha}, t\right)\leqslant\bar{t}\leqslant t+\frac{\alpha}{1-\alpha} \tag{2}$$

where t is the exact value of the standard lattice percolation conductivity exponents and $t_1=1+(d-2)\nu$ where ν is the correlation length exponent for percolation.

There have been many more recent developments in regard to continuum percolation transport properties including experimental [8], theoretical [9-11] and numerical [12, 13] investigations. The calculation of the AC conductivity exponent of a composite medium has been carried out numerically [14, 15]. Several theoretical studies of the frequency-dependent transport properties in continuum percolation systems have been described. Hui and Stroud [16] have investigated the AC response in metal-insulator composites at low frequency and in the dilute limit of metallic components using the Maxwell-Garnett (MG) approximation, and similar work has been extended near the percolation threshold ($p < p_c$) using the effective medium approximation (EMA) [17]. Both works show that at low frequency the AC conductance has an anomalous non-analytic frequency dependence $\mathrm{Re}(g_{\mathrm{eff}}) \propto \omega^{2-\alpha}$ and $\mathrm{Im}(g_{\mathrm{eff}}) \propto \omega$.

In order to confirm these theoretical predictions, we have performed a numerical simulation on a random resistor-capacitor network in which the conductances have an anomalous distribution. To our knowledge, this is the first numerical determination of the AC conductance exponents for such a system. We use a highly efficient propagation algorithm, originally proposed by Lobb and Frank [18] to reduce the square diluted resistor network to a single resistor. Such an algorithm has been applied to many problems, including the study of the surface plasmon modes of a metal-insulator composite [19] and critical current of a normal metal-superconductor composite [20]. We have generalised the algorithm to calculate the effective conductance for a long strip network. This approach was motivated by transfer matrix calculations [21, 22], which are widely used for studying transport properties on a long strip or bar lattice. As is well known, if the strip were infinitely long, the results of simulation would be exact and no ensemble average would be needed.

We have carried out our calculations on a $10 \times 20\,000$ random resistor-capacitor strip network. The conductance of the capacitor is taken as $g_2 = \mathrm{i}\omega C_0$ where $C_0 = 10^{-6}$ F. We set $\alpha = 0.4$ and resistor fraction $p = 0.05$ (which is in the dilute regime) or $p = 0.49$ (which is close to the percolation threshold $p_c = 0.5$). The calculated effective conductances for both cases are shown in figures 1 and 2 and table 1. Both figures are displayed on a log-log scale and the imaginary and real parts of effective conductance are plotted in the same figure for each value of concentration. The slopes of the curves give the exponents x and y for the power law of the frequency dependences of the imaginary and real parts of the conductances. As shown in table 1, we find that the imaginary

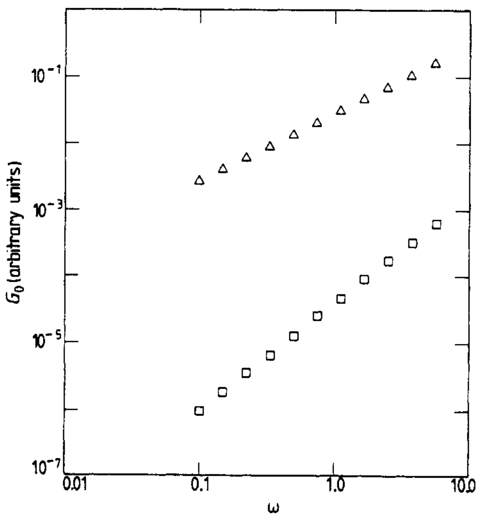

Figure 1. Real (□) and imaginary (△) parts of the AC conductance plotted as functions of frequency for a low concentration of the metallic component. The fraction of anomalous conducting bonds is $p = 0.05$ and $\alpha = 0.4$. Results are based on averaging over five realisations.

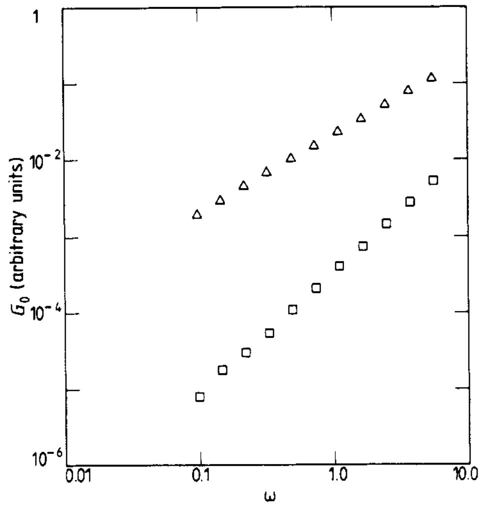

Figure 2. Real (□) and imaginary (△) parts of the AC conductance plotted as functions of frequency near the percolation threshold $p_c = 0.5$. The fraction of anomalous bonds is $p = 0.49$ and $\alpha = 0.4$. Results are based on averaging over five realisations.

Table 1. Conductance exponents x and y describing the AC conductance in a model of continuum percolation, as obtained from a least-squares fit of the data. The exponents are defined by the equations $\mathrm{Im}(g_{\mathrm{eff}}) \propto \omega^x$ and $\mathrm{Re}(g_{\mathrm{eff}}) \propto \omega^y$ at low frequencies. The calculations are carried out for $\alpha = 0.4$. p denotes the fraction of resistor bonds. Also shown are predictions of the Maxwell-Garnett and effective medium approximation.

	x	y
Simulation $p = 0.05$	0.999 ± 0.002	1.59 ± 0.01
Simulation $p = 0.49$	0.992 ± 0.002	1.58 ± 0.02
Analytic theory (MGA, EMA)	1	$2 - \alpha$

part of the conductance varies approximately as ω in each case while the real part varies approximately as $\omega^{2-\alpha}$. These results indicate that earlier theories based on the Maxwell-Garnett approximation and on the effective medium approximation give an accurate prediction of the frequency dependence for the AC conductance.

Besides simulations of the AC conductance in continuum percolation systems, we have also determined the non-universal exponent describing DC conductivity in the continuum percolation. Sen *et al* [12] have calculated such transport exponents by solving Kirchoff's equation on a small $L \times L$ lattice (with L varying from 4 to 49) and averaging over 100 realisations. Murat *et al* [13] have carried out transfer matrix calculations for a $L \times N$ strip ($5 \leqslant L \leqslant 40$, $N = 100\,000$). Since the propagation algorithm is more efficient than the previous two methods for calculating the effective conductance of a square lattice network, one can calculate the transport exponents more accurately on larger lattices and with more realisations with such an algorithm.

Finite-size scaling suggests that

$$GL/N \propto L^{-\bar{t}/\nu} \tag{3}$$

where ν is the correlation length exponent for percolation. We use (3) to numerically determine $\bar{t}$. Values of $\langle GL/N \rangle$ are calculated for several samples with L varying from 6 to 20 and $N = 20\,000$ and for samples with L from 20 to 100 and $N = 2000$. Both groups of samples are averaged over 100 realisations. The data for both cases are plotted in figure 3. The y intercept which gives the conductivity exponents is calculated by a least-squares fit of the data. Using this approach, we obtain (for $\alpha = \frac{3}{5}$) $\bar{t}/\nu = 1.90$ and $\bar{t}/\nu = 1.91$ for the two groups of samples. We can estimate the error in $\bar{t}/\nu$ by using different ranges of L to calculate it. It turns out that such statistical errors are no greater than ± 0.04. Taking $\nu = \frac{4}{3}$ (for 2D), we obtain $\bar{t} = 2.53$ and 2.55, respectively, which are within the bounds of (2). Our simulation results are thus consistent with earlier predictions and with matrix inverse and transfer matrix calculations.

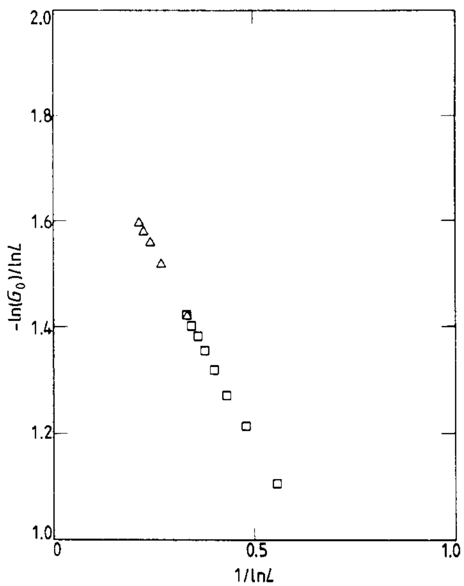

Figure 3. Plot of $-\ln(\langle GL \rangle/N)/\ln(L)$ against $1/\ln(L)$ for $\alpha = \frac{3}{5}$. According to (3), the y intercept of the line gives $\bar{t}/\nu$. Squares represent the simulation results of lattices with width L varying from 6 to 20 and length $N = 20\,000$; triangles denote the results of simulations for lattices with width L varying from 20 to 100 and length $N = 2000$.

In summary, we have calculated the AC conductance in a model of a continuum percolation system. We tested the frequency dependence of the AC conductance in the dilute limit and also near the percolation threshold. The prediction of both the Maxwell-Garnett approximation and the effective medium approximation are consistent with our simulations at low frequency. A more accurate calculation of the non-universal DC conductivity exponents is carried out in a continuum percolation network.

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